Paper Reference(s)

6667/01

Edexcel GCE

Further Pure Mathematics FP1

Advanced/Advanced Subsidiary

Monday 1 February 2010 – Afternoon

Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Mathematical Formulae (Orange)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation or integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Further Pure Mathematics FP1), the paper reference (6667), your surname, initials and signature.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. **(2)**.
There are 9 questions on this paper. The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

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1. The complex numbers z_1 and z_2 are given by

$$z_1 = 2 + 8\mathrm{i} \quad \text{and} \quad z_2 = 1 - \mathrm{i}$$

Find, showing your working,

(*a*) $\dfrac{z_1}{z_2}$ in the form $a + b\mathrm{i}$, where a and b are real,

(3)

(*b*) the value of $\left|\dfrac{z_1}{z_2}\right|$,

(2)

(*c*) the value of $\arg \dfrac{z_1}{z_2}$, giving your answer in radians to 2 decimal places.

(2)

2.

$$\mathrm{f}(x) = 3x^2 - \frac{11}{x^2}.$$

(*a*) Write down, to 3 decimal places, the value of f(1.3) and the value of f(1.4).

(1)

The equation $\mathrm{f}(x) = 0$ has a root α between 1.3 and 1.4

(*b*) Starting with the interval [1.3, 1.4], use interval bisection to find an interval of width 0.025 which contains α.

(3)

(*c*) Taking 1.4 as a first approximation to α, apply the Newton-Raphson procedure once to $\mathrm{f}(x)$ to obtain a second approximation to α, giving your answer to 3 decimal places.

(5)

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3. A sequence of numbers is defined by

$$u_1 = 2,$$

$$u_{n+1} = 5u_n - 4, \qquad n \geq 1.$$

Prove by induction that, for $n \in \mathbb{Z}$, $u_n = 5^{n-1} + 1$.

(4)

4.

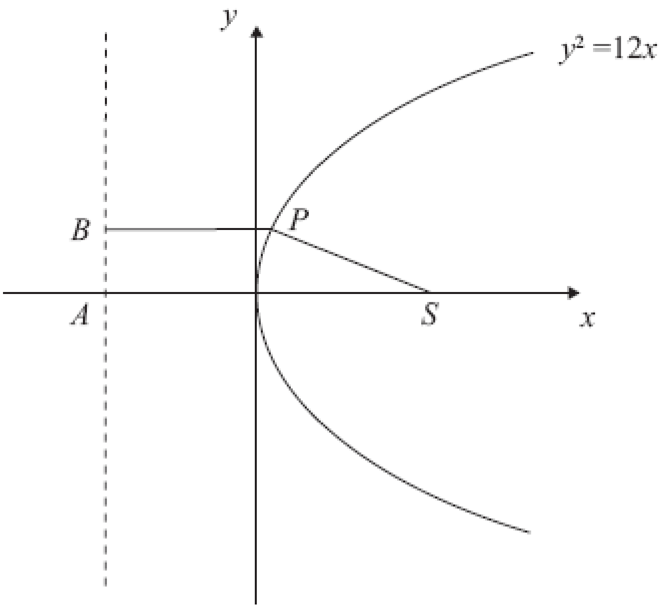

Figure 1

Figure 1 shows a sketch of part of the parabola with equation $y^2 = 12x$.

The point P on the parabola has x-coordinate $\frac{1}{3}$.

The point S is the focus of the parabola.

(a) Write down the coordinates of S.

(1)

The points A and B lie on the directrix of the parabola.
The point A is on the x-axis and the y-coordinate of B is positive.

Given that $ABPS$ is a trapezium,

(b) calculate the perimeter of $ABPS$.

(5)

5. $$\mathbf{A} = \begin{pmatrix} a & -5 \\ 2 & a+4 \end{pmatrix}, \text{ where } a \text{ is real.}$$

(*a*) Find det **A** in terms of *a*.

(2)

(*b*) Show that the matrix **A** is non-singular for all values of *a*.

(3)

Given that $a = 0$,

(*c*) find $\mathbf{A}^{-1}$.

(3)

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6. Given that 2 and 5 + 2i are roots of the equation

$$x^3 - 12x^2 + cx + d = 0, \qquad c, d \in \mathbb{R},$$

(*a*) write down the other complex root of the equation.

(1)

(*b*) Find the value of *c* and the value of *d*.

(5)

(*c*) Show the three roots of this equation on a single Argand diagram.

(2)

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7. The rectangular hyperbola *H* has equation $xy = c^2$, where *c* is a constant.

The point $P\left(ct, \dfrac{c}{t}\right)$ is a general point on *H*.

(*a*) Show that the tangent to *H* at *P* has equation

$$t^2y + x = 2ct.$$

(4)

The tangents to *H* at the points *A* and *B* meet at the point $(15c, -c)$.

(*b*) Find, in terms of *c*, the coordinates of *A* and *B*.

(5)

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8. (*a*) Prove by induction that, for any positive integer *n*,

$$\sum_{r=1}^{n} r^3 = \frac{1}{4}n^2(n+1)^2.$$

(5)

(*b*) Using the formulae for $\sum_{r=1}^{n} r$ and $\sum_{r=1}^{n} r^3$, show that

$$\sum_{r=15}^{25} (r^3 + 3r + 2) = \frac{1}{4}n(n+2)(n^2+7).$$

(5)

(*c*) Hence evaluate $\sum_{r=15}^{25} (r^3 + 3r + 2)$.

(2)

9.

$$\mathbf{M} = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}.$$

(*a*) Describe fully the geometrical transformation represented by the matrix **M**.

(2)

The transformation represented by **M** maps the point *A* with coordinates (p, q) onto the point *B* with coordinates $(3\sqrt{2}, 4\sqrt{2})$.

(*b*) Find the value of *p* and the value of *q*.

(4)

(*c*) Find, in its simplest surd form, the length *OA*, where *O* is the origin.

(2)

(*d*) Find $\mathbf{M}^2$.

(2)

The point *B* is mapped onto the point *C* by the transformation represented by $\mathbf{M}^2$.

(*e*) Find the coordinates of *C*.

(2)

TOTAL FOR PAPER: 75 MARKS

END

January 2010
6667 Further Pure Mathematics FP1
Mark Scheme

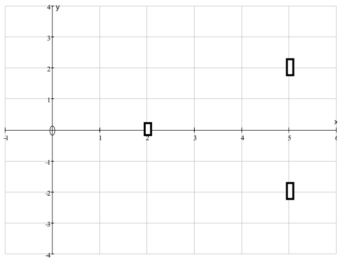
Question Number	Scheme	Marks
Q1	(a) $\dfrac{z_1}{z_2} = \dfrac{2+8\mathrm{i}}{1-\mathrm{i}} \times \dfrac{1+\mathrm{i}}{1+\mathrm{i}}$	M1
	$= \dfrac{2+2\mathrm{i}+8\mathrm{i}-8}{2} = -3+5\mathrm{i}$	A1 A1 (3)
	(b) $\left\|\dfrac{z_1}{z_2}\right\| = \sqrt{(-3)^2+5^2} = \sqrt{34}$ (or awrt 5.83)	M1 A1ft (2)
	(c) $\tan\alpha = -\dfrac{5}{3}$ or $\dfrac{5}{3}$	M1
	$\arg\dfrac{z_1}{z_2} = \pi - 1.03... = 2.11$	A1 (2) **[7]**
	Notes (a) $\times\dfrac{1+\mathrm{i}}{1+\mathrm{i}}$ and attempt to multiply out for M1 -3 for first A1, +5i for second A1 (b) Square root required without i for M1 $\dfrac{\|z_1\|}{\|z_2\|}$ award M1 for attempt at Pythagoras for both numerator and denominator (c) tan or $\tan^{-1}$, $\pm\dfrac{5}{3}$ or $\pm\dfrac{3}{5}$ seen with their 3 and 5 award M1 2.11 correct answer only award A1	

Question Number	Scheme	Marks
Q2	(a) $f(1.3) = -1.439$ and $f(1.4) = 0.268$ (allow awrt)	B1 (1)
	(b) $f(1.35) < 0$ $(-0.568...)$ $\Rightarrow$ $1.35 < \alpha < 1.4$ $f(1.375) < 0$ $(-0.146...)$ $\Rightarrow$ $1.375 < \alpha < 1.4$	M1 A1 A1 (3)
	(c) $f'(x) = 6x + 22x^{-3}$ $x_1 = x_0 - \dfrac{f(x_0)}{f'(x_0)} = 1.4 - \dfrac{0.268}{16.417}$, $= 1.384$	M1 A1 M1 A1, A1 (5) **[9]**
	Notes (a) Both answers required for B1. Accept anything that rounds to 3dp values above. (b) f(1.35) or awrt -0.6 M1 (f(1.35) and awrt -0.6) AND (f(1.375) and awrt -0.1) for first A1 $1.375 < \alpha < 1.4$ or expression using brackets or equivalent in words for second A1 (c) One term correct for M1, both correct for A1 Correct formula seen or implied and attempt to substitute for M1 awrt 16.4 for second A1 which can be implied by correct final answer awrt 1.384 correct answer only A1	

Question Number	Scheme	Marks
Q3	For n = 1: $u_1 = 2$, $u_1 = 5^0 + 1 = 2$	B1
	Assume true for n = k:	
	$u_{k+1} = 5u_k - 4 = 5(5^{k-1}+1) - 4 = 5^k + 5 - 4 = 5^k + 1$	M1 A1
	$\therefore$ True for n = k + 1 if true for n = k.	
	True for n = 1,	
	$\therefore$ true for all n.	A1 cso
		[4]
	Notes Accept $u_1 = 1 + 1 = 2$ or above B1 $5(5^{k-1}+1) - 4$ seen award M1 $5^k + 1$ or $5^{(k+1)-1} + 1$ award first A1 All three elements stated somewhere in the solution award final A1	

Question Number	Scheme	Marks
Q4	(a) (3, 0) cao	B1 (1)
	(b) P: $x = \frac{1}{3} \Rightarrow y = 2$ A and B lie on $x = -3$ $PB = PS$ or a correct method to find both PB and PS Perimeter = $6 + 2 + 3\frac{1}{3} + 3\frac{1}{3} = 14\frac{2}{3}$	B1 B1 M1 M1 A1 (5) **[6]**
	Notes (b) Both B marks can be implied by correct diagram with lengths labelled or coordinates of vertices stated. Second M1 for their four values added together. $14\frac{2}{3}$ or awrt 14.7 for final A1	

Question Number	Scheme	Marks
Q5	(a) det $\mathbf{A} = a(a+4)-(-5\times 2) = a^2+4a+10$	M1 A1 (2)
	(b) $a^2+4a+10=(a+2)^2+6$ Positive for all values of a, so $\mathbf{A}$ is non-singular	M1 A1ft A1cso (3)
	(c) $\mathbf{A}^{-1}=\frac{1}{10}\begin{pmatrix}4 & 5\\ -2 & 0\end{pmatrix}$ B1 for $\frac{1}{10}$	B1 M1 A1 (3) **[8]**
	Notes (a) Correct use of $ad-bc$ for M1 (b) Attempt to complete square for M1 Alt 1 Attempt to establish turning point (e.g. calculus, graph) M1 Minimum value 6 for A1ft Positive for all values of a, so $\mathbf{A}$ is non-singular for A1 cso Alt 2 Attempt at b^2-4ac for M1. Can be part of quadratic formula Their correct -24 for first A1 No real roots or equivalent, so $\mathbf{A}$ is non-singular for final A1cso (c) Swap leading diagonal, and change sign of other diagonal, with numbers or a for M1 Correct matrix independent of 'their $\frac{1}{10}$ award' final A1	

Question Number	Scheme	Marks
Q6	(a) $5-2\mathrm{i}$ is a root	B1 (1)
	(b) $(x-(5+2\mathrm{i}))(x-(5-2\mathrm{i}))=x^2-10x+29$ $x^3-12x^2+cx+d=(x^2-10x+29)(x-2)$ $c=49,\quad d=-58$	M1 M1 M1 A1, A1 (5)
	(c)  Conjugate pair in 1st and 4th quadrants (symmetrical about real axis) Fully correct, labelled	B1 B1 (2) **[8]**
	(b) 1st M: Form brackets using $(x-\alpha)(x-\beta)$ and expand. 2nd M: Achieve a 3-term quadratic with no i's. (b) <u>Alternative</u>: Substitute a complex root (usually 5+2i) and expand brackets — M1 $(5+2\mathrm{i})^3-12(5+2\mathrm{i})^2+c(5+2\mathrm{i})+d=0$ $(125+150\mathrm{i}-60-8\mathrm{i})-12(25+20\mathrm{i}-4)+(5c+2c\mathrm{i})+d=0$ — M1 (2nd M for achieving an expression with no powers of i) Equate real and imaginary parts — M1 $c=49,\quad d=-58$ — A1, A1	

Question Number	Scheme	Marks
Q7	(a) $y=\dfrac{c^2}{x}$ $\quad\dfrac{dy}{dx}=-c^2x^{-2}$	B1
	$\dfrac{dy}{dx}=-\dfrac{c^2}{(ct)^2}=-\dfrac{1}{t^2}$ without x or y	M1
	$y-\dfrac{c}{t}=-\dfrac{1}{t^2}(x-ct) \quad\Rightarrow\quad t^2y+x=2ct$ (*)	M1 A1cso (4)
	(b) Substitute $(15c,-c)$: $\quad -ct^2+15c=2ct$	M1
	$t^2+2t-15=0$	A1
	$(t+5)(t-3)=0 \quad\Rightarrow\quad t=-5 \quad t=3$	M1 A1
	Points are $\left(-5c,-\dfrac{c}{5}\right)$ and $\left(3c,\dfrac{c}{3}\right)$ both	A1 (5) **[9]**
	Notes (a) Use of $y-y_1=m(x-x_1)$ where m is their gradient expression in terms of c and / or t only for second M1. Accept $y=mx+k$ and attempt to find k for second M1. (b) Correct absolute factors for their constant for second M1. Accept correct use of quadratic formula for second M1. <u>Alternatives:</u> (a) $\dfrac{dx}{dt}=c$ and $\dfrac{dy}{dt}=-ct^{-2}$ B1 $\dfrac{dy}{dx}=\dfrac{dy}{dt}\div\dfrac{dx}{dt}=-\dfrac{1}{t^2}$ M1, then as in main scheme. (a) $y+x\dfrac{dy}{dx}=0$ B1 $\dfrac{dy}{dx}=-\dfrac{y}{x}=-\dfrac{1}{t^2}$ M1, then as in main scheme.	

Question Number	Scheme	Marks
Q8	(a) $\sum_{r=1}^{1} r^3 = 1^3 = 1$ and $\frac{1}{4}\times 1^2 \times 2^2 = 1$	B1
	Assume true for $n = k$:	
	$\sum_{r=1}^{k+1} r^3 = \frac{1}{4}k^2(k+1)^2 + (k+1)^3$	B1
	$\frac{1}{4}(k+1)^2\left[k^2 + 4(k+1)\right] = \frac{1}{4}(k+1)^2(k+2)^2$	M1 A1
	$\therefore$ True for $n = k + 1$ if true for $n = k$. True for $n = 1$, $\therefore$ true for all n.	A1cso (5)
	(b) $\sum r^3 + 3\sum r + \sum 2 = \frac{1}{4}n^2(n+1)^2 + 3\left(\frac{1}{2}n(n+1)\right), + 2n$	B1, B1
	$= \frac{1}{4}n\left[n(n+1)^2 + 6(n+1) + 8\right]$	M1
	$= \frac{1}{4}n\left[n^3 + 2n^2 + 7n + 14\right] = \frac{1}{4}n(n+2)(n^2+7)$ (*)	A1 A1cso (5)
	(c) $\sum_{15}^{25} = \sum_{1}^{25} - \sum_{1}^{14}$ with attempt to sub in answer to part (b)	M1
	$= \frac{1}{4}(25\times 27\times 632) - \frac{1}{4}(14\times 16\times 203) = 106650 - 11368 = 95282$	A1 (2) **[12]**
	Notes (a) Correct method to identify $(k+1)^2$ as a factor award M1 $\frac{1}{4}(k+1)^2(k+2)^2$ award first A1 All three elements stated somewhere in the solution award final A1 (b) Attempt to factorise by n for M1 $\frac{1}{4}$ and $n^3 + 2n^2 + 7n + 14$ for first A1 (c) no working 0/2	

Question Number	Scheme	Marks
Q9	(a) $45°$ or $\frac{\pi}{4}$ rotation (anticlockwise), about the origin	B1, B1 (2)
	(b) $\begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}\begin{pmatrix} p \\ q \end{pmatrix} = \begin{pmatrix} 3\sqrt{2} \\ 4\sqrt{2} \end{pmatrix}$ $p - q = 6$ and $p + q = 8$ or equivalent $p = 7$ and $q = 1$ both correct	M1 M1 A1 A1 (4)
	(c) Length of OA (= length of OB) = $\sqrt{7^2 + 1^2}$, $= \sqrt{50} = 5\sqrt{2}$	M1, A1 (2)
	(d) $\mathrm{M}^2 = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}\begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$	M1 A1 (2)
	(e) $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 3\sqrt{2} \\ 4\sqrt{2} \end{pmatrix}$ so coordinates are $(-4\sqrt{2}, 3\sqrt{2})$	M1 A1 (2) **[12]**
	Notes Order of matrix multiplication needs to be correct to award Ms (a) More than one transformation 0/2 (b) Second M1 for correct matrix multiplication to give two equations <u>Alternative</u>: (b) $\mathbf{M}^{-1} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$ First M1 A1 $\begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}\begin{pmatrix} 3\sqrt{2} \\ 4\sqrt{2} \end{pmatrix} = \begin{pmatrix} 7 \\ 1 \end{pmatrix}$ Second M1 A1 (c) Correct use of their p and their q award M1 (e) Accept column vector for final A1.	